

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Ratio

Converting between ratios, fractions and percentages

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

$$\frac{3}{4}$$

B1

[1]

Q2.

$$24 \times \frac{3}{4}$$

$$\text{or } 24 \div 4 (\times 3) \text{ or } 6 (\times 3)$$

oe

$$\text{or } 18$$

$$\text{or } 18 : 6$$

M1

$$30 : 6$$

A1

$$5 : 1$$

ft their ratio written in simplest form

B1ft

Additional Guidance

$$15 : 3 \text{ or } 10 : 2$$

M1A1B0

$$\text{answer } 1 : 5$$

M1A0B1ft

$$\text{answer } 6 : 30$$

M1A0B0ft

$$18 : 24 \text{ then } 3 : 4$$

M1A0B1ft

[3]

Q3.

$$(a) \quad 2 : 5$$

B1

$$(b) \quad \frac{3}{3+12} (\times 100) \quad \text{or} \quad \frac{3}{15} (\times 100)$$

$$\text{or} \quad \frac{1}{5} (\times 100)$$

M1

$$20$$

A1

[3]

Q4.

$$\frac{1}{3}$$

B1

[1]

Q5.

$$\frac{18}{5}$$

B1

[1]

Q6.

$$\frac{5}{14}$$

oe fraction

B1

Additional Guidance

5 : 14

B0

[1]

Q7.

$280 \div 7 \times 4$ or $280 \div 7 \times 3$ oe

If a diagram is used - rectangle split into 7 sections with 3 or 4 shaded (or indicated in some way), at least one of which has a 40 in it

M1

160 (W) or 120 (M)

A1

$180 \div 5 \times 3$ or $180 \div 5 \times 2$ oe

eg 180 - $(180 \div 5 \times 3)$

If a diagram is used – rectangle split into 5 sections with 3 or 2 shaded (or indicated in some way), at least one of which has a 36 in it.

M1

108 (B) or 72 (G)

A1

232 (F) and 228 (M) oe

*allow ft if **either** M awarded*

or 232 is more than 230 (half of 460)

or 228 is less than 230 (half of 460)

Must add men and boys and / or women and girls otherwise A0ft.

A1 ft

[5]

Q8.

$$\frac{3}{8}$$

B1

[1]

Q9.

Alternative method 1

1 part = 6 bricks

oe

M1

36 (yellow, blue and green)

A1

12 (red)

B1

$36 + 12 = 48$

B1

Alternative method 2

12 (red)

B1

$36 \div 6$ or 6

M1

their 6×2

M1dep

12 (yellow)

A1

Alternative method 3

6 parts = 75%

M1

8 parts = 100%

A1

1 part = 6 bricks

B1

$8 \times 6 = 48$

B1

[4]

Q10.

b is $\frac{3}{4}$ of a

B1

[1]

Q11.

Alternative method 1: substitutes $2f$ for d

$$\frac{e-f}{2f-e} = \frac{1}{4} \text{ or } 2f-e = 4(e-f)$$

oe equation in e and f

M1

$$6f = 5e$$

or

$$\frac{e}{f} = \frac{6}{5}$$

oe with variables collected

$$\text{eg } 1.5f = 1.25e$$

oe with single fractions eg $\frac{f}{5} = \frac{e}{6}$

M1dep

$$6:5$$

oe ratio

A1

Alternative method 2: substitutes $\frac{d}{2}$ for f

$$d-e = 4\left(e - \frac{d}{2}\right) \text{ or } 3d = 5e$$

oe equation in d and e

M1

$$6f = 5e$$

or

$$\frac{e}{f} = \frac{6}{5}$$

oe with variables collected

$$\text{eg } 1.5f = 1.25e$$

oe with single fractions eg $\frac{e}{f} = \frac{6}{5}$

M1dep

$$6:5$$

oe ratio

A1

Alternative method 3: substitutes $2f$ for d and forms simultaneous equations

$$e - f = 1$$

and

$$2f - e = 4$$

oe with rhs in the ratio 1 : 4

$$\text{eg } e - f = 2$$

and

$$2f - e = 8$$

M1

$$f = 5$$

or

$$e = 6$$

correct solution for one unknown from their correct simultaneous equations

eg $f = 10$ or $e = 12$ from above equations

M1dep

$$6 : 5$$

oe ratio

A1**Additional Guidance**

5 : 6 with no method marks awarded

M0M0A0**[3]****Q12.****Alternative method 1 – combining the ratios**

$$21 : 35 \text{ and } 35 : 20$$

$$\text{or } (3 : 5 \text{ and }) 5 : \frac{20}{7}$$

$$\text{or } \frac{21}{5} : 7 \text{ (and } 7 : 4)$$

oe making the E term common

allow as fractions with a common

$$\text{denominator eg } \frac{21}{35} \text{ and } \frac{20}{35}$$

M1

$$21 : 35 : 20$$

$$\text{or } 3 : 5 : \frac{20}{7}$$

$$\text{or } \frac{21}{5} : 7 : 4$$

$$\text{or } \frac{21/5}{76/5} \text{ or } \frac{3}{76/7}$$

oe

allow as integers 21 and 35 and 20 or as fractions with a common denominator

$$\text{eg } \frac{21}{35} \text{ and } \frac{35}{35} \text{ and } \frac{20}{35}$$

M1dep

$$\frac{21}{76}$$

A1**Alternative method 2 – based on D**

$$\frac{5(D)}{3} \text{ and } \frac{20(D)}{21}$$

oe

M1

$$\frac{21(D)}{21} + \frac{35(D)}{21} + \frac{20(D)}{21}$$

$$\text{or } \frac{76(D)}{21}$$

oe with common denominator

M1dep

$$\frac{21}{76}$$

A1

Alternative method 3 – based on E

$$\frac{3(E)}{5} \text{ and } \frac{4(E)}{7}$$

oe

M1

$$\frac{21(E)}{35} + \frac{35(E)}{35} + \frac{20(E)}{35}$$

$$\text{or } \frac{76(E)}{35}$$

oe with common denominator

M1dep

$$\frac{21}{76}$$

A1

Alternative method 4 – based on F

$$\frac{21(F)}{20} \text{ and } \frac{7(F)}{4}$$

oe

M1

$$\frac{21(F)}{20} + \frac{35(F)}{20} + \frac{20(F)}{20}$$

$$\text{or } \frac{76(F)}{20}$$

oe with common denominator

M1dep

$$\frac{21}{76}$$

A1

Additional Guidance

Allow unrounded decimal values throughout

[3]

Q13.

- (a) $23 + 9 + 20$ or 52
or 48

oe

their $48 \div 3 (\times 2)$ or 16 or 32

$23 +$ their 16 or 39
or $9 +$ their 32 or 41

39 and 41 and B

- (b) 612×4
or 612×5 or 3060

2448

M1

M1dep

M1dep

A1

M1

A1

[6]

Q14.

Total girls = 90

Girls can whistle = 36

$\frac{40}{100} \times \text{their } 90$
ft

Girls cannot whistle = 54

ft their 90 – their 36

Boys can whistle = 24

ft their $36 \div 3 \times 2$

Boys cannot whistle = 6

ft 30 – their 24

Note: if all correct B5:

B1

B1 ft

B1 ft

B1 ft

B1 ft

[5]

Q15.

$$\frac{9}{9+11} \text{ or } \frac{9}{20} \text{ or } 0.45$$

or

$$100 \div 20 \times 9 \text{ or } 5 \times 9$$

or

$$45 : 55$$

$$\text{oe eg } 9 \div 20$$

M1

$$45$$

$$SC1\ 55$$

A1

Additional Guidance

Allow eg $\frac{9}{20}$ seen with further incorrect work

$$\text{eg } \frac{9}{20} \times 11$$

M1A0

9 out of 20 with no other creditworthy work

M0

Build-up method must be a fully correct method

[2]

Q16.

$$\frac{4}{3}$$

B1

[1]

Q17.

$$\frac{1}{5}$$

B1

[1]

Q18.

$$\frac{y}{x+y}$$

B1

[1]

Q19.

$$\frac{3}{8}$$

B1

[1]

Q20.

$$\frac{1}{15}$$

oe fraction

B1

Additional Guidance

Decimal, percentage or ratio answer

B0

Do not allow 1 in 15 or 1 out of 15 unless the correct fraction seen

$$\frac{6}{100} \text{ or } \frac{7}{100} \text{ or } \frac{6.\dot{6}}{100} \text{ or } \frac{6.7}{100}$$

B0

[1]

Q21.

(a) 25(%) : 75(%)

$$\text{or } \frac{1}{4} : \frac{3}{4}$$

oe

M1

1 : 3

SC1 3 : 1

A1

(b) 19.5 ÷ 3

or 26 ÷ 4

or 6.5

oe

$$19.5 \div 75 \times 25$$

M1

6.50

Correct money notation

A1

Additional Guidance

Condone 6.50p on answer line provided £ sign is not crossed out

M1A1

[4]

Q22.

$$\frac{3}{11}$$

oe fraction

B1

[1]

Q23.

Alternative method 1

$$50 \times 1.2 \text{ or } 60$$

oe

Length of Q

May be on the diagram

M1

$$50 \times x \times 0.9 \text{ or } 45 \times x$$

oe

area of P reduced by 10%

May be on the diagram

M1

$$\text{their } 60 \times y = \text{their } 45 \times x$$

$$\text{or } \frac{y}{x} = \frac{\text{their } 45}{\text{their } 60}$$

$$\text{or } y : x = \text{their } 45 : \text{their } 60$$

or equivalent ratio to 4 : 3 not in simplest form

or equivalent fraction to $\frac{4}{3}$ not in simplest form

oe

Dep on M2

$$\text{M3 } \frac{1.2}{0.9}$$

M1dep

$$4 : 3 \text{ or } 1 : \frac{3}{4} \text{ or } 1 : 0.75 \text{ or } \frac{4}{3} : 1$$

A1

Alternative method 2

$$50 \times 1.2 \text{ or } 60$$

oe

Length of Q

May be on the diagram

M1

Chooses a value for x and reduces area of P by 10%

oe

$$\text{eg } (x = 8) 50 \times 8 \times 0.9$$

M1

Their $60 \times y =$ their area of P reduced by 10%

or equivalent ratio to 4 : 3 not in simplest form

or equivalent fraction to $\frac{4}{3}$ not in simplest form

oe

$$\text{eg } 60y = 50 \times 8 \times 0.9$$

$$\text{or } 60y = 360 \text{ or } (y =) 360 \div 60 \text{ or } 6$$

Dep on M2

$$\text{M3 } \frac{1.2}{0.9}$$

M1dep

$$4 : 3 \text{ or } 1 : \frac{3}{4} \text{ or } 1 : 0.75 \text{ or } \frac{4}{3} : 1$$

A1

Additional Guidance

Allow 1.33(...) for $\frac{4}{3}$

4 : 3 in working with 3 : 4 on answer line

M3A0

$$1 : \frac{45}{60}$$

M3A0

(Alt 1) $50x = 60y \times 0.9$

M1M0M0A0

(Alt 1) $50x = 60y \times 1.1$

M1M0M0A0

(Alt 1) $45x : 60y$

M1M1

Answer 3 : 4

M0A0

(Alt 1) $y : x = 3 : 4$

Answer 3 : 4

M3A0

Alt 2 example

$50 \times 10 = 500$ (working not seen for reduction by 10% but completed correctly in next line)

$450 \div 60 = 7.5$ (60 here gains first M1)

M1M1

$10 : 7.5 = 20 : 15$

M1A0

Do not allow misreads

eg increases length of P by 10% (instead of 20%)

Alt 2 Allow choice of x to be 50

[4]**Q24.**

(a) $\frac{392}{7} \times 2$

oe

M1

112

SC1 504

A1

(b) $\frac{8}{11}$ or 0.72... or 0.73

oe or 72(...)% or 73%

B1**[3]**

Q25.

Alternative method 1

$72 \div 6 \times 5$ or 60 oe
 $72 \div 6 \times 11$ or 132 implies M1

M1

72×1.5 or 108 oe eg $72 \times 3 \div 2$
 14×12 implies M2

M1

60 and 108 and 240
or $250 - 60 - 108 = 82$
oe eg1 168 and 240
eg2 60 and 108 and 10
eg3 168 and $(250 - 72 =) 178$

A1

Alternative method 2

6×1.5 or 9
oe eg1 $6 \times 3 \div 2$
eg2 $6 : 5 : 9$

M1

$72 \div 6 \times (6 + 5 + \text{their } 9)$
or $72 \div 6 \times 5$ and $72 \div 6 \times \text{their } 9$
oe eg 12×20
 14×12 implies M2

M1dep

9 and 240
or 60 and 108 and 240
or $250 - 60 - 108 = 82$
oe eg1 168 and 240
eg2 60 and 108 and 10
eg3 168 and $(250 - 72 =) 178$

A1

Additional Guidance

Up to M2 may be awarded for correct work, with no or incorrect answer, even if this is seen amongst multiple attempts

In Alt 1 the 2nd mark is not dependent

In Alt 2 the 2nd mark is dependent

240 alone or 240 with no correct method

M0

$72 \div 6 \times 11 = 132$ and $132 + 108 = 240$

M1M1A1

$1\frac{1}{2} \times 72 = 36$ and $72 + 36 = 108$ and $72 + 60 + 108 = 240$

M1M1A1

$1\frac{1}{2} \times 72 = 36$

M1

$1\frac{1}{2}$ of 72 = 36

M0

$72 \div 11$

M0

[3]

Q26.

$$\frac{3}{8} \times 120 \text{ or } 45$$

oe

may be implied or embedded

$$\text{eg } \left(1 - \frac{3}{8}\right) \times 120 \text{ or } \frac{5}{8} \times 120 \text{ or } 75$$

M1

$$(120 - \text{their } 45) \div (2 + 1)$$

or

$$75 \div 3 \text{ or } 25$$

oe

may be implied or embedded

$$\text{eg } 75 \times \frac{2}{3} \text{ or } \frac{5}{8} \times 120 \times \frac{2}{3}$$

M1dep

$$50$$

A1

Additional Guidance

M1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts

25 : 50 on answer line

M1M1A0

[3]

Q27.

$$\frac{100 - 68}{2} \text{ or } \frac{32}{2} \text{ or } 16(\%)$$

or

$$\frac{1 - 0.68}{2} \text{ or } \frac{0.32}{2} \text{ or } 0.16$$

oe

M1

$$68 : 16$$

oe ratio not in form $n : 1$

eg $68\% : 16\%$ or $17 : 4$ or $0.68 : 0.16$

or

$$\frac{68}{16} \text{ or } 68 \div 16 \text{ or } 4.25$$

oe fraction or division or decimal
implied by $4.25 : 1$ oe

A1

$$4.25 : 1 \text{ or } 4\frac{1}{4} : 1$$

oe ratio in form $n : 1$ eg $\frac{68}{16} : 1$
ft any **ratio** not in form $n : 1$
ft values must give n to 2 dp or better

Additional Guidance

$$\frac{100 - 68}{2} = 66$$

M1

$$68 : 66 = 1.03 : 1$$

A0B1ft

$$68 : 32 = 2.125 : 1 \text{ or } 68 : 32 = 2.13 : 1$$

M0A0B1ft

$$68 \div 32, \text{ Answer } 2.125 : 1 \text{ (no ratio seen to ft)}$$

M0A0B0ft

Correct ratio with subsequent truncation or rounding to < 2 dp
eg1 $4.25 : 1$, Answer $4 : 1$

M1A1B0

$$\text{eg2 } 68 : 32 = 2.125 : 1, \text{ Answer } 2.1 : 1$$

M0A0B0

$$4.25n : 1$$

M1A1B0

$$16 : 1 \text{ with no other creditworthy work}$$

M1A0B0

[3]

Q28.

35 : 21 and 21 : 12

or

5 : 3 : $\frac{12}{7}$ or $\frac{35}{7}$: $\frac{21}{7}$: $\frac{12}{7}$

or

$\frac{35}{3}$: 7 : 4 : or $\frac{35}{3}$: $\frac{21}{3}$: $\frac{12}{3}$

Any correct pair of ratios where the values for women are equal

or

a correct three-part ratio

M1

their 35 + their 21 + their 12 or 68

or their 21 + their 12 or 33

Could be multiples of these numbers

M1dep

$35 \div 68 = 0.51\dots$ or $51\dots\%$

or

35 and (half of 68 is) 34

or

35 (men) and 33 (women and children)

oe

A1

[3]

Q29.

Alternative method 1

$$4a = 9b$$

$$\text{oe} \\ \frac{a}{b} = \frac{9}{4}$$

M1

$$4a = 9 \times \frac{7c}{10}$$

$$\text{or } 40a = 63c$$

$$40a = 90b$$

$$\text{and } 90b = 63c$$

$$\text{oe} \\ 9 : \frac{40}{7}$$

M1dep

$$63 : 40$$

$$\text{Accept } \frac{63}{40} : 1 \text{ or } 1.575 : 1 \\ \text{or } 1 : \frac{40}{63}$$

A1

Alternative method 2

$$b : c = 7 : 10$$

M1

$$a : b = 63 : 90 \text{ and } b : c = 90 : 40 \text{ or } 63 : 90 : 40$$

oe common value for b

M1dep

$$63 : 40$$

$$\text{Accept } \frac{63}{40} : 1 \text{ or } 1.575 : 1 \\ \text{or } 1 : \frac{40}{63}$$

A1

Alternative method 3

$$a = \frac{9b}{4} \text{ or } c = \frac{10b}{7}$$

M1

$$\frac{9b}{4} : \frac{10b}{7} \text{ or } \frac{9}{4} : \frac{10}{7}$$

oe

M1dep

$$63 : 40$$

$$\text{Accept } \frac{63}{40} : 1 \text{ or } 1.575 : 1 \\ \text{or } 1 : \frac{40}{63}$$

A1

Alternative method 4

$$c = \frac{10}{7}b$$

$$\text{eg } a : c = a : \frac{10}{7}b$$

M1

$$9 : \frac{10}{7} \times 4 \text{ or } 9 : \frac{40}{7}$$

oe

M1dep

63 : 40

$$\text{Accept } \frac{63}{40} : 1 \text{ or } 1.575 : 1$$

$$\text{or } 1 : \frac{40}{63}$$

A1**Additional Guidance**

2nd method mark is for a link between a and c or a correct ratio in an unsimplified form

40 : 63 on answer line

M1M1A0**[3]****Q30.**

$$250 \div 5 \text{ or } 50$$

$$\text{or } 250 \times 3 \text{ or } 750$$

$$\text{or } 250 \times 2 \div 5 \text{ or } 100$$

$$\text{or } 0.12 \times 250 \text{ or } 30 \quad \text{oe}$$

M1

$$\frac{250 \times 3}{5}$$

$$\text{or } 150$$

$$\text{or } 250 - \text{their } 100 \text{ or } 150$$

$$\text{or their } 30 \div 5 \text{ or } 6$$

$$\text{or their } 30 \times 3 \text{ or } 90 \quad \text{oe}$$

M1

$$\frac{250 \times 3}{5}$$

$$0.12 \times 5$$

$$\text{or } 0.12 \times 150$$

$$\text{or their } 30 \div 5 \times 3 \quad \text{oe}$$

M1

18

A1

20

ft 360 ÷ their 18 (rounded up if non integer)

B1ft**Additional Guidance**

For their final answer, if their 20 is not a whole number then must round answer up correctly to get the final B mark

For those who work out 88% of 150 giving an answer of 132 they will score M1M1 for the 150

150, 18, 132, 3 weeks

M1M1M0A0B1ft

150, 18, 132

M1M1M0A0B0**[5]**